SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Write the partial fraction decomposition of the rational expression.

1) $\frac{9x+8}{(x-6)^2}$ 1) __________

Write the form of the decomposition: $\frac{9x+8}{(x-6)^2} = \frac{A}{x-6} + \frac{B}{(x-6)^2}$

Multiply both sides by $(x-6)^2$: $9x+8 = A(x-6)+B$

Simplify: $9x+8 = Ax + (-6A+B)$

To find A and B, equate the coefficients on both sides of the equal sign and solve:

$A = 9$

$-6A + B = 8$
$-6(9) + B = 8$
$B = 62$

So, the partial fraction decomposition is:

$$\frac{9x+8}{(x-6)^2} = \frac{9}{x-6} + \frac{62}{(x-6)^2}$$

2) $\frac{32-5x}{x^3-8x^2+16x}$ 2) __________

$$\frac{32-5x}{x^3-8x^2+16x} = \frac{32-5x}{x(x^2-8x+16)} = \frac{32-5x}{x(x-4)^2} = \frac{-5x+32}{x(x-4)^2}$$

Write the form of the decomposition: $\frac{-5x+32}{x(x-4)^2} = \frac{A}{x} + \frac{B}{x-4} + \frac{C}{(x-4)^2}$

Multiply both sides by $x(x-4)^2$: $-5x+32 = A(x-4)^2 + Bx(x-4) + Cx$

Expand and simplify: $-5x+32 = A(x^2-8x+16) + B(x^2-4x) + Cx$

$-5x+32 = (A+B)x^2 + (-8A-4B+C)x + 16A$

To find A, B and C, equate the coefficients on both sides of the equal sign and solve:

$A+B=0$ $-8A-4B+C$ $16A=-5$

Solve for A:
$16A = 32$
$\boldsymbol{A = 2}$

Then, solve for B:
$A+B=0$
$2+B=0$
$\boldsymbol{B=-2}$

Then, solve for C:
$-8A-4B+C=-5$
$-8(2)-4(-2)+C=-5$
$-16+8+C=-5$
$\boldsymbol{C=3}$

So, the partial fraction decomposition is:

$$\frac{32-5x}{x(x-4)^2} = \frac{2}{x} + \frac{-2}{x-4} + \frac{3}{(x-4)^2}$$

Write the form of the partial fraction decomposition of the rational expression. It is not necessary to solve for the constants.

3) $\dfrac{6x+2}{(x-7)(x^2+x+3)^2}$ 3) ____________

$$\frac{6x+2}{(x-7)(x^2+x+3)^2}$$

Note that x^2+x+3 cannot be factored because its discriminant is negative.
That is, $\Delta = b^2 - 4ac = 1^2 - (4 \cdot 1 \cdot 3) < 0.$

If x^2+x+3 could be factored, we would need to factor it before setting up the form of the partial fraction decomposition. Watch for this on the test.

Write the form of the decomposition: $\frac{6x+2}{(x-7)(x^2+x+3)^2} = \frac{A}{x-7} + \frac{Bx+C}{(x^2+x+3)} + \frac{Dx+E}{(x^2+x+3)^2}$

I'm glad we don't have to solve this one. It would give a system of 5 equations in 5 unknowns. Yuk!

Solve the system by the substitution method.

4) $x^2 + y^2 = 113$
$x + y = 15$ 4) ____________

$x^2 + y^2 = 113 \qquad x + y = 15$

$x = 15 - y$

$(15-y)^2 + y^2 = 113$

$(y^2 - 30y + 225) + y^2 = 113$

$2y^2 - 30y + 112 = 0$

$y^2 - 15y + 56 = 0$

$(y-7)(y-8) = 0$

$y = \{7, 8\}$

When $y = 7$, we get:

$x + 7 = 15,$ so $x = 8 \quad \Rightarrow \quad (8, 7)$ is a solution

When $y = 8$, we get:

$x + 8 = 15,$ so $x = 7 \quad \Rightarrow \quad (7, 8)$ is a solution

So, our solutions are: $\{(\mathbf{7}, \mathbf{8}), (\mathbf{8}, \mathbf{7})\}$

5) xy = 12
$x^2 + y^2 = 40$

5) ____________

$x^2 + y^2 = 40$ $\qquad$ $xy = 12$

$y = \dfrac{12}{x}$ $\quad$ note that $x \neq 0$ since $xy = 12$, so we can do this.

$$x^2 + \left(\frac{12}{x}\right)^2 = 40$$

$$x^2 + \frac{144}{x^2} = 40$$

$x^4 + 144 = 40x^2$ $\qquad$ (after multiplying both sides by x^2)

$$x^4 - 40x^2 + 144 = 0$$

$$(x^2 - 4)(x^2 - 36) = 0$$

$$(x + 2)(x - 2)(x + 6)(x - 6) = 0$$

$$x = \{\pm 2, \pm 6\}$$

Let's use the equation $y = \dfrac{12}{x}$ to find the y-values for each x-value.

When $x = -2$, we get:

$y = \dfrac{12}{-2} = -6 \Rightarrow$ $(-2, -6)$ is a solution

When $x = 2$, we get:

$y = \dfrac{12}{2} = 6 \Rightarrow$ $(2, 6)$ is a solution

When $x = -6$, we get:

$y = \dfrac{12}{-6} = -2 \Rightarrow$ $(-6, -2)$ is a solution

When $x = 6$, we get:

$y = \dfrac{12}{6} = 2 \Rightarrow$ $(6, 2)$ is a solution

So, our solutions are: $\{(-2, -6), (2, 6), (-6, -2), (6, 2)\}$

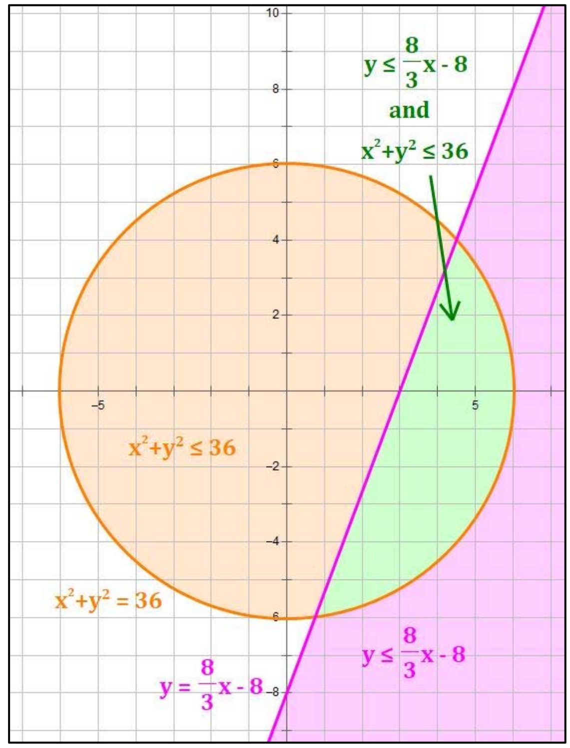
Graph the solution set of the system of inequalities or indicate that the system has no solution.

6) $\begin{cases} -8x + 3y \le -24 \\ x^2 + y^2 \le 36 \end{cases}$

8) ______________

$\boldsymbol{x^2 + y^2 \le 36}$ (orange and green areas)

- Graph the circle: $x^2 + y^2 = 36$.
- Some points on the curve: $(0,6), (0,-6), (6,0), (-6,0)$
- The curve will be solid because there is an "equal sign" included in the inequality.
- Fill in the interior of the circle because of the "less than" sign in the inequality.



$\boldsymbol{-8x + 3y \le -24}$ (violet and green areas)

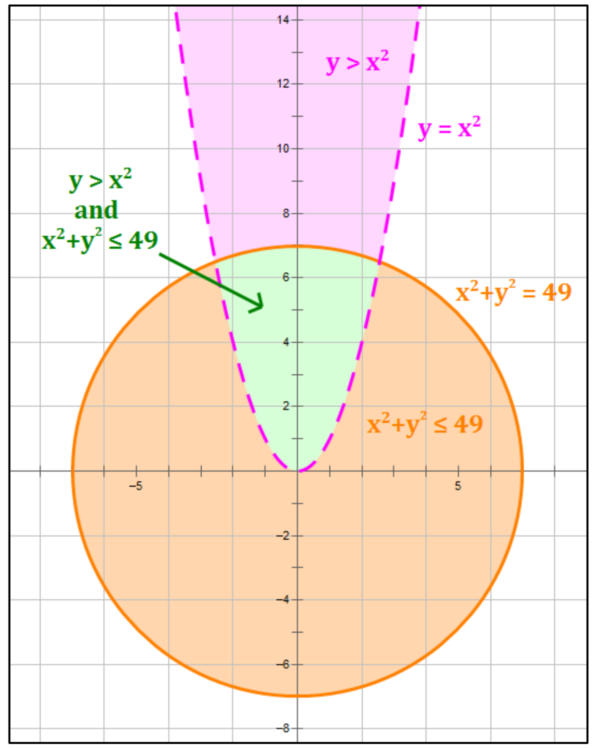
- Put this in "$y \le mx + b$" form
  $3y \le 8x - 24$
  $y \le \frac{8}{3}x - 8$
- Graph the line: $y = \frac{8}{3}x - 8$.
- Some points on the line: $(0,-8), (3,0)$
- The line will be solid because there is an "equal sign" included in the inequality.
- Fill in the portion of the graph below the curve because of the "less than" sign in the inequality.

The green shaded area is the area of intersection of the given inequalities.

7) $\begin{cases} y - x^2 > 0 \\ x^2 + y^2 \leq 49 \end{cases}$

9) ______________

$\boldsymbol{x^2 + y^2 \leq 49}$ (orange and green areas)



- Graph the circle: $x^2 + y^2 = 49$.
- Some points on the curve:
  $(0, 7), (0, -7), (7, 0), (-7, 0)$
- The curve will be solid because there is an "equal sign" included in the inequality.
- Fill in the interior of the circle because of the "less than" portion of the inequality.

$\boldsymbol{y - x^2 > 0}$ (violet and green areas)

- Put this in "$y >$" form
  $y - x^2 > 0$
  $y > x^2$
- Graph the parabola: $y = x^2$.
- Some points on the curve:
  $(0, 0), (2, 4), (-2, 4)$
- The curve will be dashed because there is no "equal sign" included in the inequality.
- Fill in the portion of the graph above the curve because of the "greater than" sign in the inequality.

The green shaded area is the area of intersection of the given inequalities.

8) Graph the constraints and use the objective function to maximize the function.

Objective function: $z = 23x + 8y$
Constraints: $0 \le x \le 10$
$0 \le y \le 5$
$3x + 2y \ge 6$

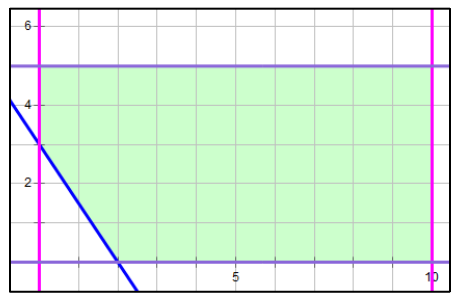
We will need to graph the constraints to find the points of intersection. The maximum and minimum values of the objective function will be at these points.

Constraints: $0 \le x \le 10$ $\quad$ $0 \le y \le 5$ $\quad$ $3x + 2y \ge 6$

$y \ge -\frac{3}{2}x + 3$

Points of intersection (based on the graph): $(2,0), (0,3), (0,5), (10,0), (10,5)$

We are instructed in the statement of the problem to check the Objective Function value (OFV) at each point of intersection, even though **it is obvious that the point $(10, 5)$ will maximize** the OFV because of it's position relative to other points on the graph.

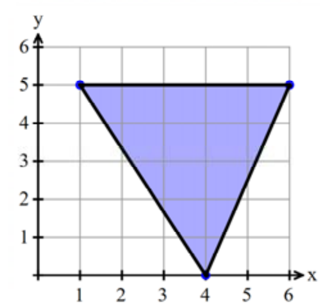


Objective Function values:

Point $(2,0)$: $z = 23x + 8y = 23(2) + 8(0) = 46$
Point $(0,3)$: $z = 23x + 8y = 23(0) + 8(3) = 24$
Point $(0,5)$: $z = 23x + 8y = 23(0) + 8(5) = 40$
Point $(10,0)$: $z = 23x + 8y = 23(10) + 8(0) = 230$
Point $(10,5)$: $z = 23x + 8y = 23(10) + 8(5) = 270$

The maximum value of the Objective Function occurs at $(10, 5)$ and is equal to 270.

9) Given the graph shown on the right, which shows the feasible region of a linear program problem. If the objective function is $z = 3x + 4y$, then what is the maximum value?



Let's check the Objective Function value at each vertex.

Objective Function values:

Point $(1,5)$: $z = 3x + 4y = 3(1) + 4(5) = 23$
Point $(6,5)$: $z = 3x + 4y = 3(6) + 4(5) = 38$
Point $(4,0)$: $z = 3x + 4y = 3(4) + 4(0) = 12$

The maximum value of the Objective Function occurs at $(6, 5)$ and is equal to 38.

10) Write the equation of an ellipse in standard form that meets the requirements below:
foci: $(0,-2),(0,2)$; y-intercepts: -5 and 5.

This ellipse has foci $(0,\pm 2)$, which are on the y-axis. Therefore, the ellipse has a vertical major axis.

The standard form for an ellipse with a vertical major axis is:

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

Remember that $a>b$ for an ellipse. That's why a^2 is in the denominator of the y-term.

Note: graphs of conic sections for problems in this packet were made with the Algebra (Main) App, available at: www.mathguy.us/PCApps.php.

The values of a and b can be determined from the foci and the y-intercepts.

The center of the ellipse is halfway between the foci, i.e., at $(0,0)$, so $(h,k)=(0,0)$.

The foci are located $c=2$ units from the center. So, we have determined that:

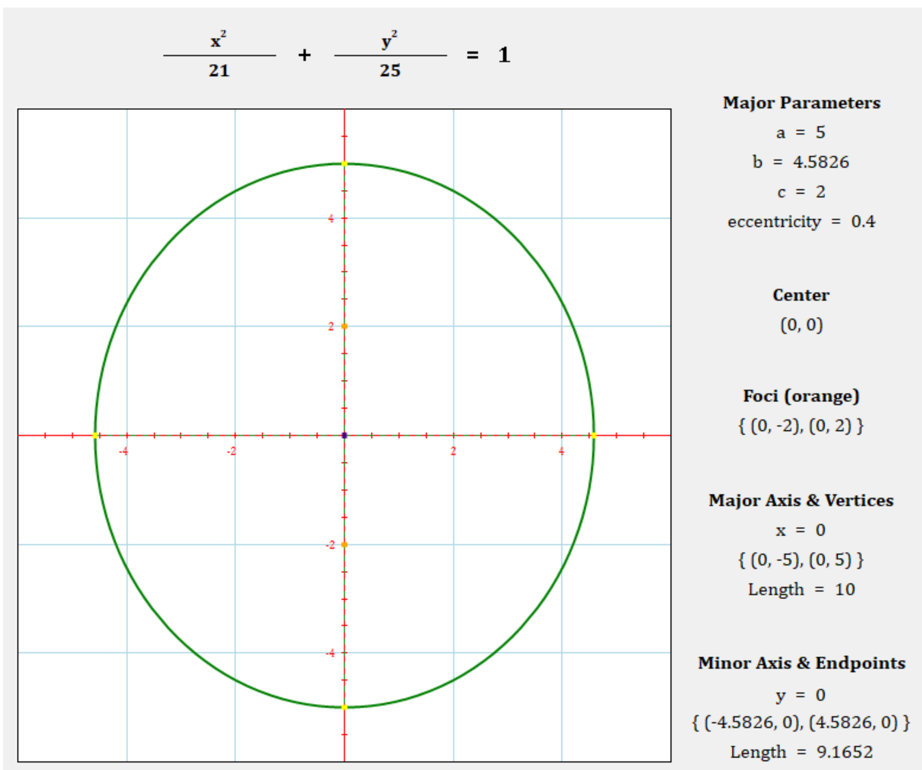
- $h=0,\ k=0,\ c=2$

The y-intercepts, then, are major axis vertices, which are located $a=5$ units from the center. So, we have determined that:

- $a=5 \quad\Rightarrow\quad a^2=25$
- $c^2=a^2-b^2$ for an ellipse, so: $2^2=5^2-b^2$, giving $b^2=21$

Then, substituting values into the standard form equation gives:

$$\frac{x^2}{21}+\frac{y^2}{25}=1$$



11) Write the equation of a hyperbola in standard form that meets the requirements below:
foci: $(0,-4),(0,4)$; vertices: $(0,-3),(0,3)$.

This hyperbola has foci $(0,\pm 4)$, and therefore has a vertical transverse axis.

The standard form for a hyperbola with a vertical transverse axis is:

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

Remember that a^2 is in the denominator of the lead term for a hyperbola.

The values of a and b can be determined from the foci and the vertices.

The center of the hyperbola is halfway between the foci, i.e., at $(0,0)$, so $(h,k)=(0,0)$.

The foci are located $c=4$ units from the center. So, we have determined that:

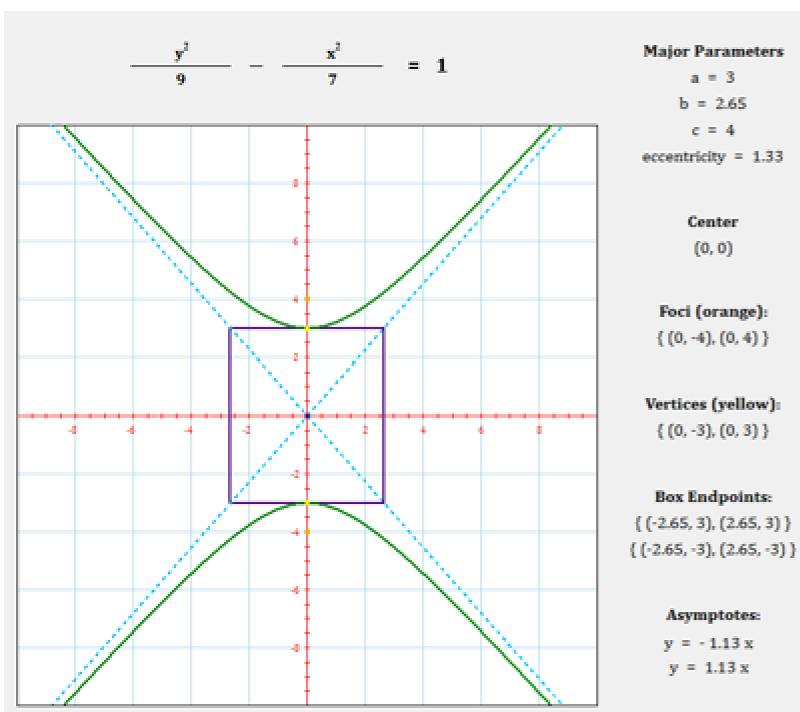
- $h=0,\ k=0,\ c=4$

The vertices are located $a=3$ units from the center. So, we have determined that:

- $a=3 \quad \Rightarrow \quad a^2=9$
- $c^2=a^2+b^2$ for a hyperbola, so $4^2=3^2+b^2$, giving $b^2=7$

Then, substituting values into the standard form equation gives:

$$\frac{y^2}{9}-\frac{x^2}{7}=1$$



For #12 – 14, graph the conic and find the requested information. If needed, round to 3 decimal places.

12) $4x^2 + 8x + 9y^2 = 32$ (Convert to standard form by completing the square)

$$4x^2 + 8x + 9y^2 = 32$$

$$(4x^2 + 8x + \quad\;) + 9y^2 = 32$$

$$4(x^2 + 2x + \quad\;) + 9y^2 = 32$$

$$4(x^2 + 2x + 1) + 9y^2 = 32 + 4(1)$$

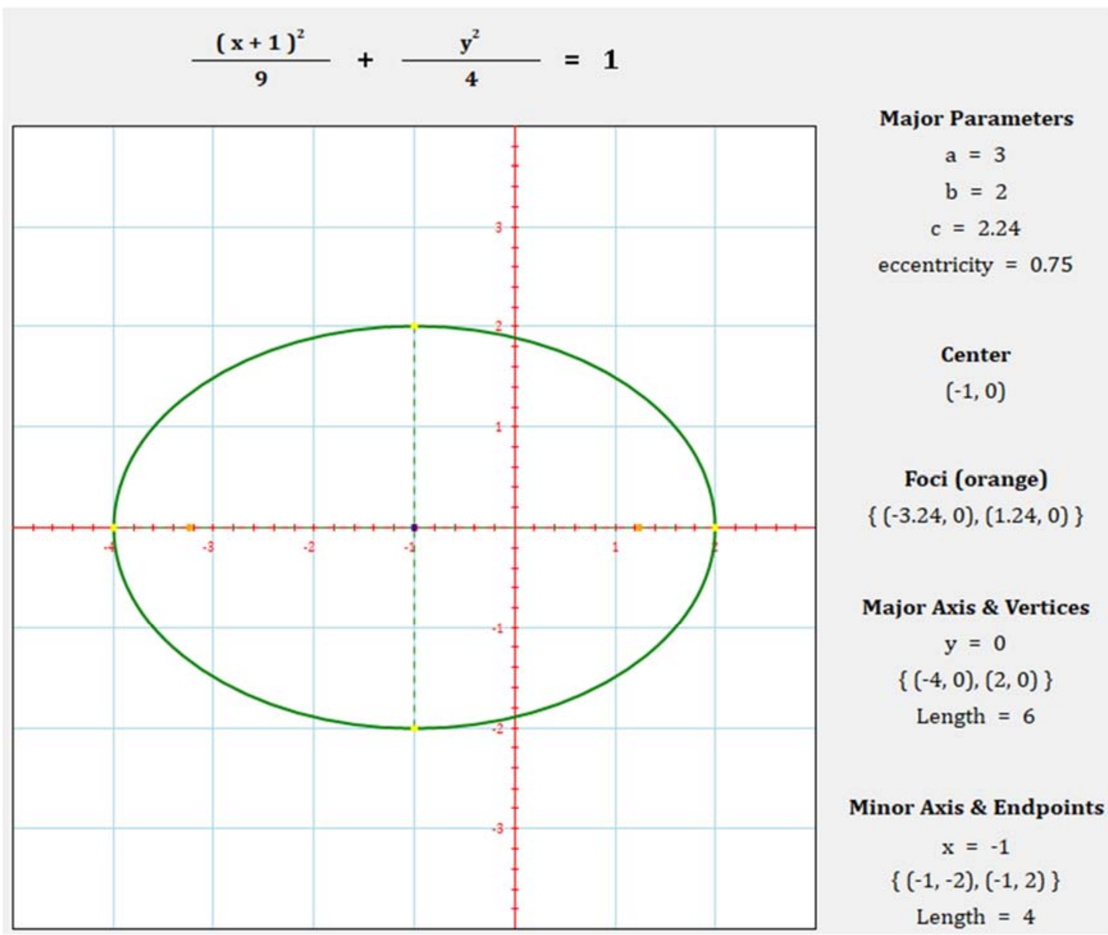
$$4(x + 1)^2 + 9y^2 = 36$$

$$\frac{(x+1)^2}{9} + \frac{y^2}{4} = 1$$

Center: $(-1, 0)$

The foci are $\sqrt{9 - 4} = \sqrt{5} \sim 2.236$ away from the center along the major axis, i.e., in both x-directions (left and right).

So, the **foci** are: $(-1 - 2.236, 0) = (-3.236, 0)$ and $(-1 + 2.236, 0) = (1.236, 0)$.



13) $x^2 - 2x + 7y - 34 = 0$ (Convert to standard form by completing the square)

The key to graphing a parabola is to identify its vertex and orientation (which way it opens). Consider the form of the above equation:

$$(x-h)^2 = 4p(y-k)$$

$$x^2 - 2x + 7y - 34 = 0$$

$$(x^2 - 2x + \_\_\_\_) = -7y + 34$$

$$(x^2 - 2x + 1) = -7y + 34 + 1$$

$$(\boldsymbol{x} - \mathbf{1})^2 = -\mathbf{7}(\boldsymbol{y} - \mathbf{5})$$

From this equation, we can determine the following:

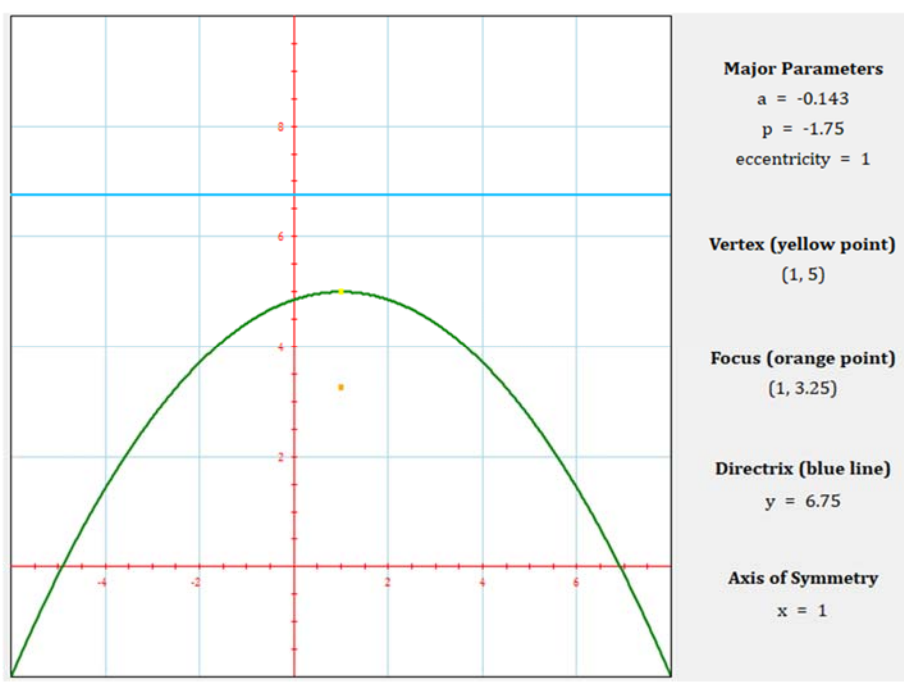
- The **vertex** of the parabola is $(h, k) = (\mathbf{1}, \mathbf{5})$.
- Since the x-term is squared, the parabola has a horizontal Directrix (i.e., it opens up or down).
- The **length of the latus rectum** is $|4p| = \mathbf{7}$. $p = -7/4$ is negative, so the parabola opens down.
- Since the x-term is squared, the **focus** is located $7/4$ units below the vertex, $(1, 5)$: $(\mathbf{1}, \mathbf{3.25})$.
- The **equation of the directrix** is: $y = k - p \quad \Rightarrow \quad y = 5 - \left(-\frac{7}{4}\right) \quad \Rightarrow \quad \boldsymbol{y} = \mathbf{6.75}$

Let's find a couple of points to help us draw our graph of the parabola. Rewrite the equation in a simpler form to find y, given x.

$$y = -\frac{1}{7}(x-1)^2 + 5$$

We already have a point – the vertex, at $(1, 5)$. Let's find a couple more:

- Let $x = -2$. Then $-\frac{1}{7}(-2-1)^2 + 5 = 3\frac{5}{7}$. This gives us the point $\left(-2, 3\frac{5}{7}\right)$.
- Let $x = 4$. Then $-\frac{1}{7}(4-1)^2 + 5 = 3\frac{5}{7}$. This gives us the point $\left(4, 3\frac{5}{7}\right)$.



14) $\frac{(x-2)^2}{16} - \frac{(y+2)^2}{4} = 1$

This equation is already in standard form.

$\frac{(x-2)^2}{16} - \frac{(y+2)^2}{4} = 1$ Standard form is: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ or $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

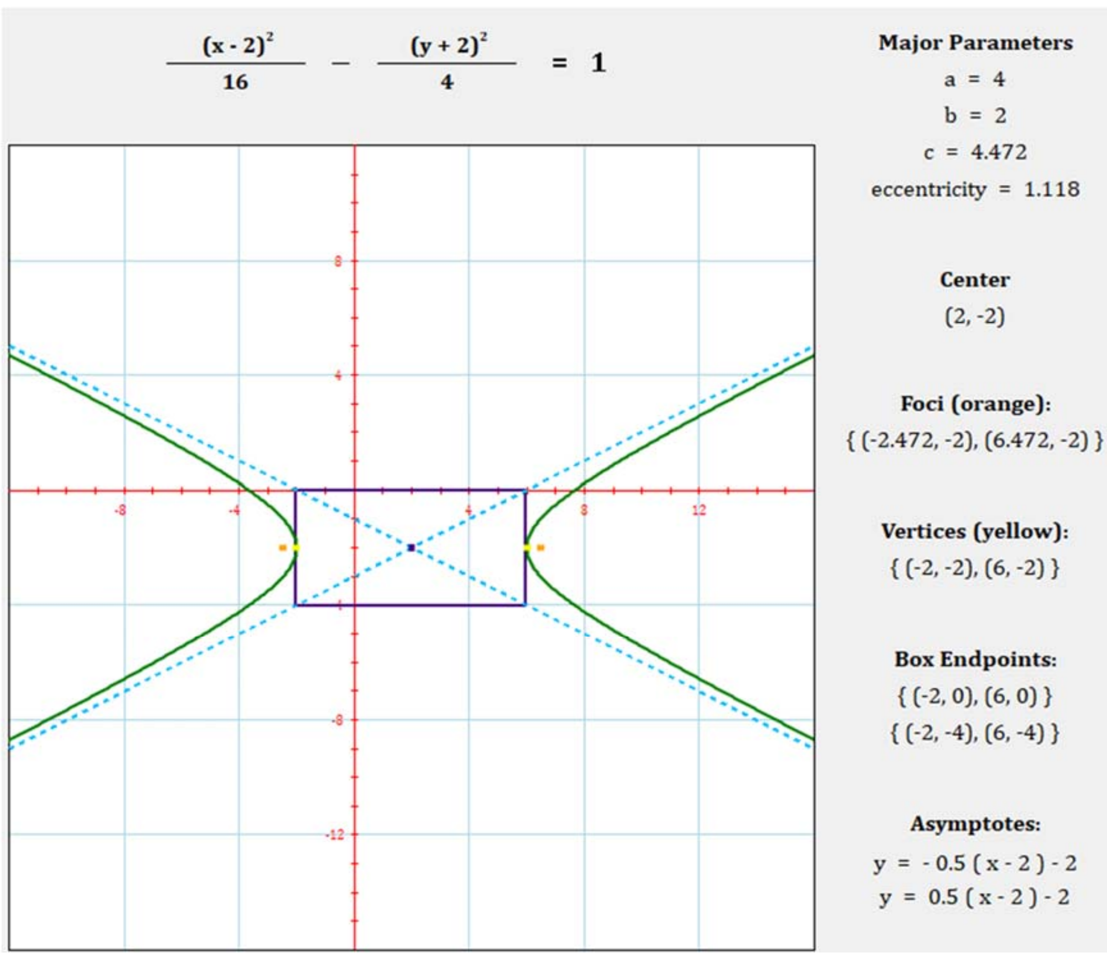
Since the x-term is positive, we know that the hyperbola has a horizontal transverse axis.

- $h = 2,\ k = -2$
- $a^2 = 16$, so $a = 4$, $b^2 = 4$, so $b = 2$, $c^2 = a^2 + b^2 = 16 + 4 = 20$, so $c = \sqrt{20} \sim 4.472$
- **The center is** (h, k): $(\mathbf{2}, \mathbf{-2})$
- The center, vertices, and foci lie on the horizontal transverse axis (HTA). On an HTA:
  The vertices are a units left and right from the center: $(2 \pm 4, -2) \Rightarrow \{(-2,-2), (6,-2)\}$
  The foci are c units left and right from the center: $(2 \pm \sqrt{20}, 0) \Rightarrow \{(\mathbf{-2.472}, \mathbf{0}), (\mathbf{6.472}, \mathbf{0})\}$

The asymptotes are not required, but for a hyperbola with a horizontal transverse axis are: $y = \pm\frac{b}{a}(x-h) + k$.

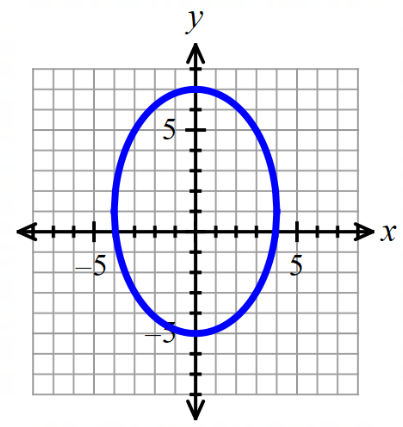
The asymptotes are: $y = \pm\frac{b}{a}(x-h) + k \Rightarrow y = \pm\frac{1}{2}(x-2) - 2$

To graph the hyperbola, first graph the asymptotes and the vertices, and then sketch in the rest.



For #15 – 20, match each graph to its equation. No item will be used more than once. Not all equations will be used.

15)



An **ellipse** has a $+$ sign between terms and different denominators in the two terms.

The center is $(0, 1)$.

Each denominator is the squares of half of the length of its axis.

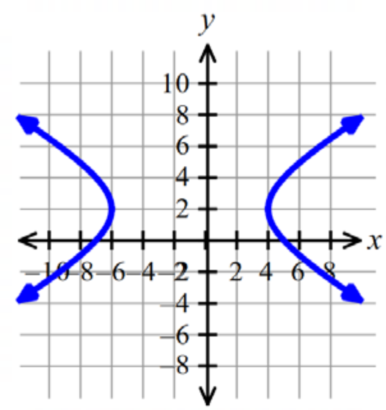
x-axis length is 8, so the x-denominator is $\left(\frac{8}{2}\right)^2 = 16$.

y-axis length is 12, so the y-denominator is $\left(\frac{12}{2}\right)^2 = 36$.

The equation, then, is:

$$\frac{x^2}{16} + \frac{(y-1)^2}{36} = 1 \quad \textbf{Answer D}$$

16)



A **hyperbola** has a $-$ sign between terms.

It opens left and right if the x-term is positive.

Its vertices are $(-6, 2), (4, 2)$.

Its center is halfway between the vertices: $(-1, 2)$.

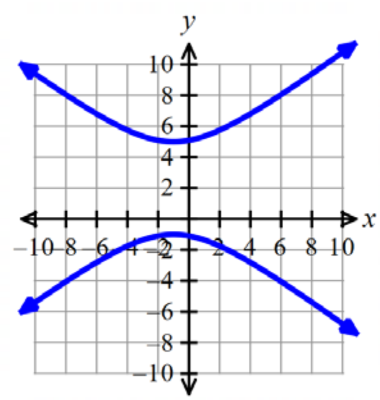
Its horizontal transverse axis has length: $4 - (-6) = 10$. The value of a is half this length, $a = 5$, and so $a^2 = 25$.

This hyperbola, then, has an equation of the form:

$$\frac{(x+1)^2}{25} - \frac{(y-2)^2}{b^2} = 1$$

Answers A and B are hyperbolas. **Answer B** has the proper form.

17)



A **hyperbola** has a $-$ sign between terms.

It opens up and down if the y-term is positive.

Its vertices are $(-1, -5), (-1, 1)$.

Its center is halfway between the vertices: $(-1, 2)$.

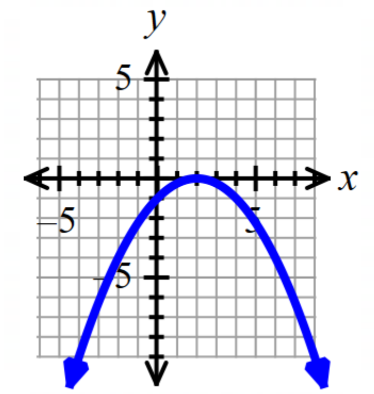
Its vertical transverse axis has length: $1 - (-5) = 6$. The value of a is half this length, $a = 3$, and so $a^2 = 9$.

This hyperbola, then, has an equation of the form:

$$\frac{(y-2)^2}{9} - \frac{(x+1)^2}{b^2} = 1$$

Answers A and B are hyperbolas. **Answer A** has the proper form.

18)



A **parabola** has only one of the terms squared.

This parabola opens down (in the y-direction), so p is negative and the squared term is the x-term.

The vertex is $(2, 0)$.

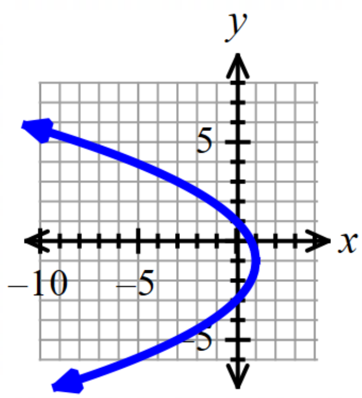
The equation must have the form:

$(\boldsymbol{x-2})^{\boldsymbol{2}} = \boldsymbol{-cy}$, with c a constant.

Answers C and F are parabolas.

Answer F has the proper form.

19)



A **parabola** has only one of the terms squared.

This parabola opens to the left (in the x-direction), so p is negative and the squared term is the y-term.

The vertex is $(1, -1)$.

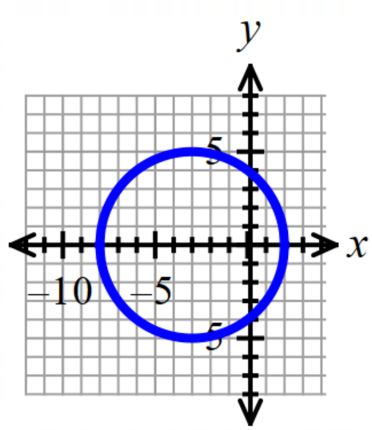
The equation must have the form:

$(\boldsymbol{y+1})^{\boldsymbol{2}} = \boldsymbol{-c(x-1)}$, with c a constant.

Answers C and F are parabolas.

Answer C has the proper form.

20)



A **circle** has a $+$ sign between terms, and no (or the same) denominators.

The center of this circle is $(-3, 0)$.

The radius of the circle is $r = 5$, so $r^2 = 25$.

The equation must have the form:

$(\boldsymbol{x+3})^{\boldsymbol{2}} + \boldsymbol{y}^{\boldsymbol{2}} = \boldsymbol{25}$.

Answers F, G, and H are circles.

Answer H is correct.

Write a formula for the general term (the nth term) of the arithmetic sequence. Then use the formula for a_n to find a_{20}, the 20th term of the sequence.

21) 11, 4 , −3, −10, . . . 19) ____________

Unorthodox, but I like to find $a_0 = a_1 - d$ because $\boldsymbol{a_0}$ **(i.e., the 0th term) is the constant term** in the explicit formula for a_n and $\boldsymbol{d}$ **is the multiplier of** $\boldsymbol{n}$. So, the explicit formula for an arithmetic sequence is always:

$a_n = a_0 + dn$ (note: you need to calculate a_0; it is not given)

For this sequence, $d = 4 - 11 = \mathbf{-7}$, and so $a_0 = a_1 - d = 11 - (-7) = \mathbf{18}$

Then, the explicit formula is: $\boldsymbol{a_n = 18 - 7n}$

Finally, $\boldsymbol{a_{20}} = 18 - 7(20) = 18 - 140 = \mathbf{-122}$

Find the indicated sum.

22) Find the sum of the first 20 terms of the arithmetic sequence: −12, −6, 0, 6, . . . 20) ____________

Method 1: Think like Gauss

First, we need: $a_{20} = a_1 + 19d = -12 + 19(6) = 102.$ Then:

$$\begin{aligned} S &= -12 \quad -6 \quad +0 \quad +6 + \cdots + 102 \\ S &= \ 102 + 96 + 90 + 84 + \cdots \ -12 \\ \hline 2S &= \ \ 90 + 90 + 90 + 90 + \cdots \ +90 = 20(90) \end{aligned}$$

Divide both sides by 2, to get

$S = 10(90) = \mathbf{900}$

Method 2: Use the arithmetic series sum formula: $S = \left(\frac{n}{2}\right) \cdot (a_1 + a_n)$

Again, we need $a_{20} = a_1 + 19d = -12 + 19(6) = 102$

$a_1 = -12 \qquad a_{20} = 102 \qquad n = 20$

$$S = \left(\frac{n}{2}\right) \cdot (a_1 + a_n) = \left(\frac{20}{2}\right) \cdot (-12 + 102) = 10(90) = \mathbf{900}$$

Use the formula for the general term (the nth term) of a geometric sequence to find the indicated term of the sequence with the given first term, a_1, and common ratio, r.

23) Find a_{12} when $a_1 = -5$, $r = 2$. 21) ____________

The general term of a geometric sequence is: $a_n = a_1 \cdot r^{n-1}$

$a_1 = -5$ $\qquad r = 2$

Then, $a_n = -5 \cdot (2)^{n-1}$

So, $a_{12} = -5 \cdot (2)^{12-1} = -5 \cdot 2048 = \mathbf{-10,240}$

Write a formula for the general term (the nth term) of the geometric sequence.

24) $3, -\frac{3}{2}, \frac{3}{4}, -\frac{3}{8}, \frac{3}{16}, \ldots$ 22) ____________

The general term of a geometric sequence is: $a_n = a_1 \cdot r^{n-1}$

$a_1 = 3$ $\qquad r = \dfrac{-\frac{3}{2}}{3} = \dfrac{-3}{2 \cdot 3} = -\dfrac{1}{2}$

Then, $\boldsymbol{a_n = 3 \cdot \left(-\frac{1}{2}\right)^{n-1}}$

25) Find the sum of the geometric sequence: $\sum_{n=1}^{8}(-4)^n$

$$x = \sum_{n=1}^{8}(-4)^n = -4 + 16 - 64 + 256 - 1024 + 4096 - 16384 + 65536$$

$$+4x = \quad -16 + 64 - 256 + 1024 - 4096 + 16384 - 65536 + 262144$$

$$5x = -4 + 262144$$

$$5x = 262{,}140$$

$$\boldsymbol{x = 52{,}428}$$

26) To save for retirement, you decide to deposit \$2250 into an IRA at the end of each month for the next 35 years, with an interest rate of 5% compounded monthly. Find the value of the IRA after 35 years. Round to the nearest dollar.

Note: I don't know what "Round to the nearest dollar if needed" means. I don't think we "need" to.

Let's look at the series that results from this. Note: deposits are made at the **end** of the month.

The first month's deposit will earn $\frac{5}{12}$% per month for 419 months.

The second month's deposit will earn $\frac{5}{12}$% per month for 418 months.

…

The final month's deposit will earn $\frac{5}{12}$% per month for 0 months.

Then, $S = 2{,}250 \cdot \left[\left(1.+\frac{.05}{12}\right)^{419} + \left(1.+\frac{.05}{12}\right)^{418} + \left(1.+\frac{.05}{12}\right)^{417} + \cdots + 1\right]$

Look at the series inside the brackets in reverse order:

$$\left(1+\frac{.05}{12}\right)x = \left[\quad \left(1+\frac{.05}{12}\right)^{1} + \left(1+\frac{.05}{12}\right)^{2} + \cdots + \left(1+\frac{.05}{12}\right)^{419} + \left(1+\frac{.05}{12}\right)^{420}\right]$$

$$- \qquad x = \left[1 + \left(1+\frac{.05}{12}\right)^{1} + \left(1+\frac{.05}{12}\right)^{2} + \cdots + \left(1+\frac{.05}{12}\right)^{419}\right]$$

$$\frac{.05}{12}x = \left[-1 \qquad\qquad + \left(1+\frac{.05}{12}\right)^{420}\right]$$

$$x = \frac{\left(1+\frac{.05}{12}\right)^{420} - 1}{\left(\frac{.05}{12}\right)} = 1136.09243$$

Accumulated Value $= \$2{,}250 \cdot 1{,}136.09243 \sim \$2{,}556{,}208$

Alternative, using the formula:

$a_1 = 2{,}250 \quad r = \left(1+\frac{.05}{12}\right) \quad n = 420$

Calculation is the same as above:

$$S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right) = 2{,}250 \cdot \left(\frac{\left(1+\frac{.05}{12}\right)^{420} - 1}{\left(\frac{.05}{12}\right)}\right) \sim \$2{,}556{,}208$$

Note: \$2,250 per month is way above federal guidelines on how much can be deposited into an IRA, so don't get too excited.

Find the sum of the infinite geometric series, if it exists.

27) $96 + 24 + 6 + \frac{3}{2} + \ldots$ 26) ____________

Method 1: Think like Gauss

$a_1 = 96 \qquad r = \frac{1}{4}$

$$\begin{aligned} S &= 96 + 24 + 6 + \frac{3}{2} + \cdots \\ -\frac{1}{4}S &= \quad -24 - 6 - \frac{3}{2} - \cdots \\ \hline \frac{3}{4}S &= 96 \end{aligned}$$

Note that this series converges because: $|r| = \left|\frac{1}{4}\right| = \frac{1}{4} < 1$.

Multiply both sides by $\frac{4}{3}$, to get

$$S = \frac{4}{3} \cdot 96 = \mathbf{128}$$

Method 2: Use the infinite geometric series sum formula: $S = a_1 \cdot \left(\frac{1}{1-r}\right)$

$a_1 = 96 \qquad r = \frac{1}{4}$

$$S = a_1 \cdot \left(\frac{1}{1-r}\right) = 96\left(\frac{1}{1-\left(\frac{1}{4}\right)}\right) = \frac{96}{\frac{3}{4}} = \frac{96}{1} \cdot \frac{4}{3} = \mathbf{128}$$

28) $\frac{1}{3} - 1 + 3 - \cdots$

This is a Geometric Series with $r = -3$. Note that:

$$\frac{a_2}{a_1} = \frac{-1}{\frac{1}{3}} = -1 \cdot \frac{3}{1} = -3 \qquad \text{and} \qquad \frac{a_3}{a_2} = \frac{3}{-1} = -3$$

A Geometric Series converges if $|r| < 1$ and diverges otherwise. Therefore, **this series diverges**. So **the sum does not exist**.

Express the repeating decimal as a fraction in lowest terms.

29) $0.\overline{58}$ 29) ____________

Let $x = 0.\overline{58}$. Then think like our old buddy, Gauss.

$$\begin{aligned} 100x &= 58.\overline{58} \\ -x &= -0.\overline{58}. \\ \hline 99x &= 58 \quad \Rightarrow \quad x = \mathbf{\frac{58}{99}} \end{aligned}$$

Remember the shortcut is to place the repeating digits in the numerator, and the same number of 9's in the denominator.

30) Write the equation of a parabola in standard form that meets the requirements below: Focus at $(-4,5)$ and directrix at $x=-2$.

The parabola described above has a vertical Directrix, so it opens left or right.

The focus is to the left of the Directrix, so the parabola opens to the left.

For a parabola with a vertical Directrix:

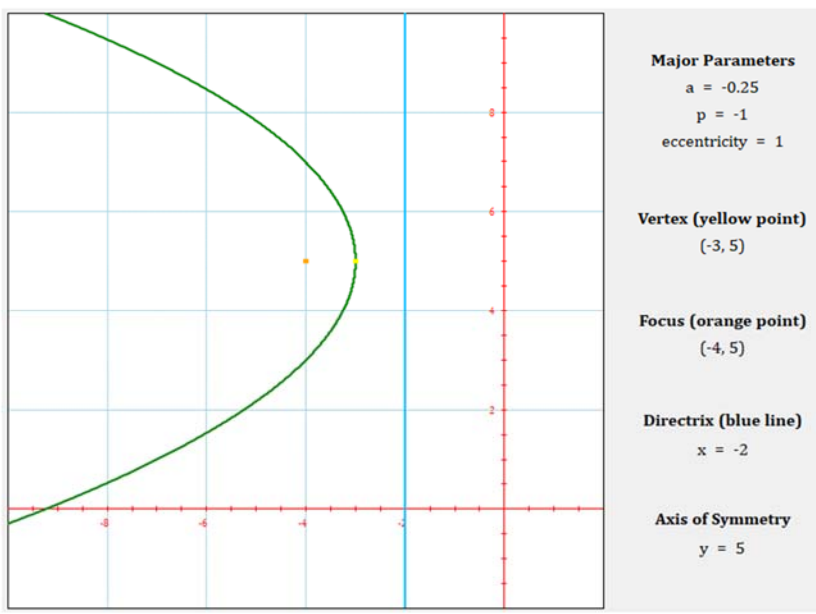
- The vertex is halfway between the focus and the Directrix, so the vertex is:

$$(h,k)=\left(\frac{-4+(-2)}{2},5\right)=(-3,5) \quad \Rightarrow \quad h=-3;\ k=5$$

- p is negative because the parabola opens to the left.
- $|p|$ is the distance from the vertex to the focus. $p=-|-4-(-3)|=-1$

We can now write the equation in standard form: $(y-k)^2=4p(x-h)$

$$(y-5)^2=-4(x-3)$$



For 31 – 35 , verify the trig identity:

31) $\tan x(\cot x-\cos x)=1-\sin x$

$$\tan x\cdot(\cot x-\cos x) \qquad = 1-\sin x$$

$$\frac{\sin x}{\cos x}\cdot\left(\frac{\cos x}{\sin x}-\frac{\cos x}{1}\right)$$

$$\left(\frac{\sin x}{\cos x}\cdot\frac{\cos x}{\sin x}\right)-\left(\frac{\sin x}{\cos x}\cdot\frac{\cos x}{1}\right)$$

$$\mathbf{1} \quad - \quad \mathbf{\sin x} \qquad = \mathbf{1-\sin x} \quad ✓$$

32) $\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$

$$\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} \qquad = \tan\alpha + \tan B$$

$$\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}$$

$$\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}$$

$$\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}$$

$$\boldsymbol{\tan\alpha + \tan B} \qquad \boldsymbol{= \tan\alpha + \tan B} \qquad \checkmark$$

33) $\frac{1+\cos 2x}{\sin 2x} = \cot x$

$$\frac{1+\cos 2x}{\sin 2x} \qquad = \cot x$$

$$\frac{1 + (2\cos^2 x - 1)}{2\sin x\cos x}$$

$$\frac{2\cos^2 x}{2\sin x\cos x}$$

$$\frac{\cos x}{\sin x}$$

$$\boldsymbol{\cot x} \qquad \boldsymbol{= \cot x} \qquad \checkmark$$

34) $\sin\left(x + \frac{\pi}{2}\right) = \cos x$

$$\sin\left(x + \frac{\pi}{2}\right) \qquad = \cos x$$

$$\sin x\cos\frac{\pi}{2} + \cos x\sin\frac{\pi}{2}$$

$$(\sin x)\cdot 0 + (\cos x)\cdot 1$$

$$\boldsymbol{\cos x} \qquad \boldsymbol{= \cos x} \qquad \checkmark$$

35) $\tan^2\theta + 4 = \sec^2\theta + 3$

$$\tan^2\theta + 4 \qquad = \sec^2\theta + 3$$

$$(\tan^2\theta + 1) + 3$$

$$\boldsymbol{\sec^2\theta + 3} \qquad \boldsymbol{= \sec^2\theta + 3} \qquad \checkmark$$

For 36 – 37: Find the exact value by using a sum or difference identity.

36) $\tan\frac{7\pi}{12}$

$$\tan\frac{7\pi}{12} = \tan\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$$

$$= \frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{3}}{1 - \left(\tan\frac{\pi}{4}\right)\left(\tan\frac{\pi}{3}\right)}$$

$$= \frac{1+\sqrt{3}}{1 - 1\cdot\sqrt{3}} = \frac{1+\sqrt{3}}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{1+2\sqrt{3}+3}{1-3} = \frac{4+2\sqrt{3}}{-2} = \boldsymbol{-2-\sqrt{3}}$$

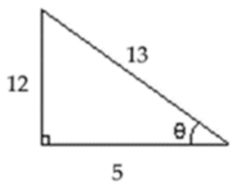
37) $\cos\frac{11\pi}{12}$

$$\cos\frac{11\pi}{12} = \cos\left(\frac{3\pi}{12} + \frac{8\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right)$$

$$= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{2\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{2\pi}{3}\right)$$

$$= \frac{\sqrt{2}}{2}\left(-\frac{1}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) = \boldsymbol{\frac{-\sqrt{2}-\sqrt{6}}{4}} = \boldsymbol{-\frac{\sqrt{2}+\sqrt{6}}{4}}$$

38) Use the figure to find the exact value of $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$.

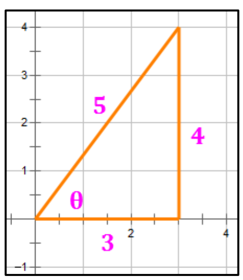


$$\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{12}{13}\cdot\frac{5}{13} = \mathbf{\frac{120}{169}}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2 = \frac{25-144}{169} = \mathbf{\frac{-119}{169}}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\frac{120}{169}}{\frac{-119}{169}} = -\mathbf{\frac{120}{119}}$$

39) Use the given information to find the exact value of $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$.
$\sin\theta = \frac{4}{5}$, θ lies in quadrant I



$$\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{4}{5}\cdot\frac{3}{5} = \mathbf{\frac{24}{25}}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = -\mathbf{\frac{7}{25}}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = -\mathbf{\frac{24}{7}}$$

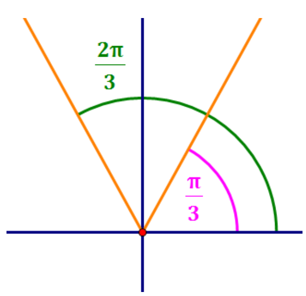
For 40 – 41, find all solutions of the following equations.

40) $2\sin x - \sqrt{3} = 0$

$$2\sin x - \sqrt{3} = 0$$

$$2\sin x = \sqrt{3}$$

$$\sin x = \frac{\sqrt{3}}{2}$$



The drawing at left illustrates the two angles in $[0, 2\pi)$ for which $\sin x = \frac{\sqrt{3}}{2}$. To get all solutions, we need to add all integer multiples of 2π to these solutions. So,

$$\boldsymbol{x \in \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\}}$$

41) $\tan x \sec x = -2 \tan x$

$\tan x \sec x = -2 \tan x$

$\tan x \sec x + 2 \tan x = 0$

$\tan x\ (\sec x + 2) = 0$

$\tan x = 0$ or $(\sec x + 2) = 0$

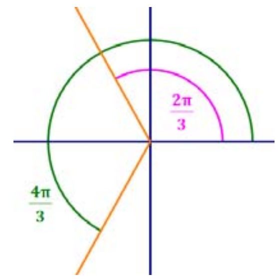
$x = 0 + n\pi = n\pi$

$(\sec x + 2) = 0$

$\sec x = -2$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3} + 2n\pi$ or $x = \frac{4\pi}{3} + 2n\pi$



Collecting the various solutions, $\boldsymbol{x \in \{n\pi\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\}}$

Note: the solution involving the tangent function has two answers in the interval $[0, 2\pi)$. However, they are π radians apart, as most solutions involving the tangent function are. Therefore, we can simplify the answers by showing only one base answer and adding $n\pi$, instead of showing two base answers that are π apart, and adding $2n\pi$ to each.

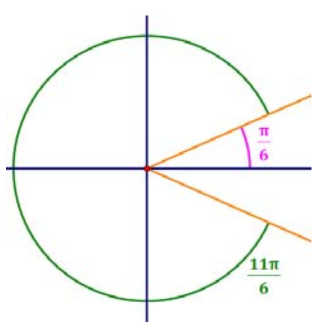
For example, the following two solutions for $\tan x = 0$ are telescoped into the single solution given above:

$$\left.\begin{aligned} x &= 0 + 2n\pi = \{\dots, -4\pi, -2\pi, 0, 2\pi, 4\pi, \dots\} \\ x &= \pi + 2n\pi = \{\dots, -3\pi, -\pi, \pi, 3\pi, 5\pi \dots\} \end{aligned}\right\} \quad x = 0 + n\pi = \{\dots, -2\pi, -\pi, 0, \pi, 2\pi, \dots\}$$

For 42 – 45, solve the equation on the interval $[0, 2\pi)$.

42) $\cos 2x = \frac{\sqrt{3}}{2}$

We want all solutions to $\cos u = \frac{\sqrt{3}}{2}$ where $u = 2x$ is an angle in the interval $[0, 4\pi)$. Note that, beyond the two solutions suggested by the diagram, additional solutions are obtained by adding 2π to those two solutions.



Note that there are 4 solutions because the usual number of solutions (i.e., 2) is increased by a factor of $k = 2$.

Using the diagram at left, we get the following solutions:

$$u = 2x = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}$$

Then, dividing by 2, we get:

$$\boldsymbol{x = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}}$$

We cannot simplify these solutions any further.

43) $\cos^2 x + 2\cos x + 1 = 0$

The trick on this problem is to replace the trigonometric function, in this case, $\cos x$, with a variable, like u, that will make it easier to see how to factor the expression. If you can see how to factor the expression without the trick, by all means proceed without it.

Let $u = \cos x$, and our equation becomes: $u^2 + 2u + 1 = 0$.

This equation factors to get: $(u+1)^2 = 0$

Substituting $\cos x$ back in for u gives: $(\cos x + 1)^2 = 0$

And finally: $\cos x + 1 = 0 \quad \Rightarrow \quad \cos x = -1$

The only solution for this on the interval $[0, 2\pi)$ is: $\boldsymbol{x = \pi}$

44) $\cos x + 2\cos x \sin x = 0$

$\cos x + 2\cos x \sin x = 0$

$\cos x\,(1 + 2\sin x) = 0$

$\cos x = 0$ or $(1 + 2\sin x) = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$(1 + 2\sin x) = 0$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

Collecting the various solutions, $\boldsymbol{x = \left\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}}$

45) $\cos\left(x + \frac{\pi}{3}\right) + \cos\left(x - \frac{\pi}{3}\right) = 1$

The following formulas will help us solve this problem.

$\cos\ (\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\qquad$ $\cos\ (\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\cos\left(x + \frac{\pi}{3}\right) + \cos\left(x - \frac{\pi}{3}\right) = 1$$

$$\cos x \cos\frac{\pi}{3} - \sin x \sin\frac{\pi}{3} + \cos x \cos\frac{\pi}{3} + \sin x \sin\frac{\pi}{3} = 1$$

$$2\cos x \cos\frac{\pi}{3} = 1$$

$$2\cos x \cdot \frac{1}{2} = 1$$

$$\cos x = 1$$

$$\boldsymbol{x = 0}$$

46) Evaluate the given binomial coefficient: $\binom{10}{5}$

$$\binom{10}{5} = \frac{10!}{5! \cdot (10-5)!} = \frac{10!}{5! \cdot 5!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \mathbf{252}$$

47) Use the Binomial Theorem to expand the binomial and express the result in simplified terms: $(2x-1)^5$

General Formula:

$$(\boldsymbol{a}+\boldsymbol{b})^{\boldsymbol{n}} = \sum_{\boldsymbol{k}=0}^{\boldsymbol{n}} \binom{\boldsymbol{n}}{\boldsymbol{k}} \boldsymbol{a}^{\boldsymbol{n}-\boldsymbol{k}} \boldsymbol{b}^{\boldsymbol{k}}$$

Step 1: Start with the binomial coefficients

$$\binom{5}{0} + \binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5}$$

Step 2: Add in the powers of the first term of the binomial $(2x)$

$$\binom{5}{0}(2x)^5 + \binom{5}{1}(2x)^4 + \binom{5}{2}(2x)^3 + \binom{5}{3}(2x)^2 + \binom{5}{4}(2x)^1 + \binom{5}{5}(2x)^0$$

Step 3: Add in the powers of the second term of the binomial (-1)

$$\binom{5}{0}(2x)^5(-1)^0 + \binom{5}{1}(2x)^4(-1)^1 + \binom{5}{2}(2x)^3(-1)^2 + \binom{5}{3}(2x)^2(-1)^3 + \binom{5}{4}(2x)^1(-1)^4 + \binom{5}{5}(2x)^0(-1)^5$$

Step 4: Simplify:

$$= (1)(32x^5)(1) + (5)(16x^4)(-1) + (10)(8x^3)(1) + (10)(4x^2)(-1) + (5)(2x)(1) + (1)(1)(-1)$$

$$= \mathbf{32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x - 1}$$

48) Find the 8th term in the following binomial expansion: $(x - 3y)^{11}$

General Formula:

$$(\boldsymbol{a} + \boldsymbol{b})^{\boldsymbol{n}} = \sum_{\boldsymbol{k}=\boldsymbol{0}}^{\boldsymbol{n}} \binom{\boldsymbol{n}}{\boldsymbol{k}} \boldsymbol{a}^{\boldsymbol{n}-\boldsymbol{k}} \boldsymbol{b}^{\boldsymbol{k}}$$

KEY POINT: Unfortunately, there are several ways to answer this question, based on how the "8th term" is defined. In order to be consistent with the Pearson textbook and homework problems, we must set the value of k to be one less than the number of the term. Using this approach, the first term has $k = 0$, so the 8th term has $k = 7$. Other sources name the terms differently.

The terms of the binomial expansion of $(a + b)^n$ are typically given by the formula:

$$\binom{n}{k} a^{n-k} b^k$$

Then, using the approach described above for this problem:

$a = x \qquad b = -3y \qquad n = 11 \qquad term = 8 \qquad k = 7$

And, so,

$$\binom{n}{k} a^{n-k} b^k = \binom{11}{7} (x)^{11-7} (-3y)^7 = \frac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2 \cdot 1} (x)^4 (-3y)^7 = \mathbf{-721710}\, \boldsymbol{x^4 y^7}$$

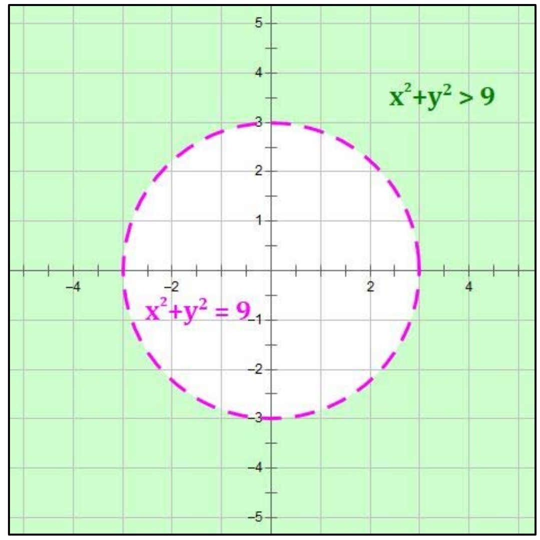
Additional problems on prior reviews:

Graph the inequality.

6) $x^2 + y^2 > 9$ 6) ______________

$\boldsymbol{x^2 + y^2 > 9}$ (green area in the diagram)

- Graph the circle: $x^2 + y^2 = 9$.
- Some points on the curve: $(0, 3), (0, -3), (3, 0), (-3, 0)$
- The curve will be dashed because there is no "equal sign" included in the inequality.
- Fill in the exterior of the circle because of the "greater than" sign in the inequality.



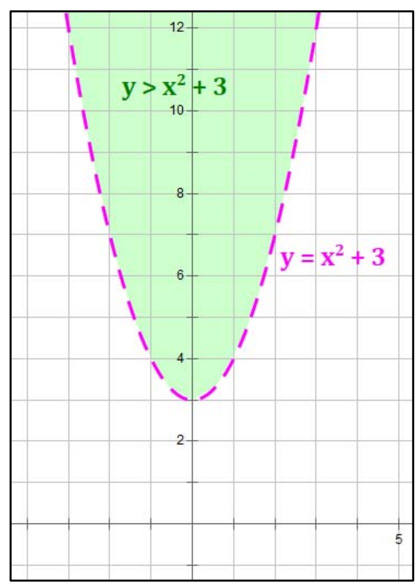
The green shaded area is the area required.

7) $y > x^2 + 3$ 7) ______________

-

$\boldsymbol{y > x^2 + 3}$ (green area in the diagram)

- Graph the parabola: $y = x^2+3$.
- Some points on the curve: $(0, 3), (2, 7), (-2, 7)$
- The curve will be dashed because there is no "equal sign" included in the inequality.
- Fill in the portion of the graph above the curve because of the "greater than" sign in the inequality.



The green shaded area is the area required.

13) Endpoints of major axis: (−2, 1) and (−2, 7); endpoints of minor axis: (−4, 4) and (0, 4);   13) ____________

Since the major axis endpoints have the same x-value, the ellipse has a vertical major axis.

An ellipse with a vertical major axis has the following characteristics:

- Standard form is: $\dfrac{(x-h)^2}{b^2}+\dfrac{(y-k)^2}{a^2}=1$
- The center is at (h,k), which is the midpoint of the major axis vertices, so:

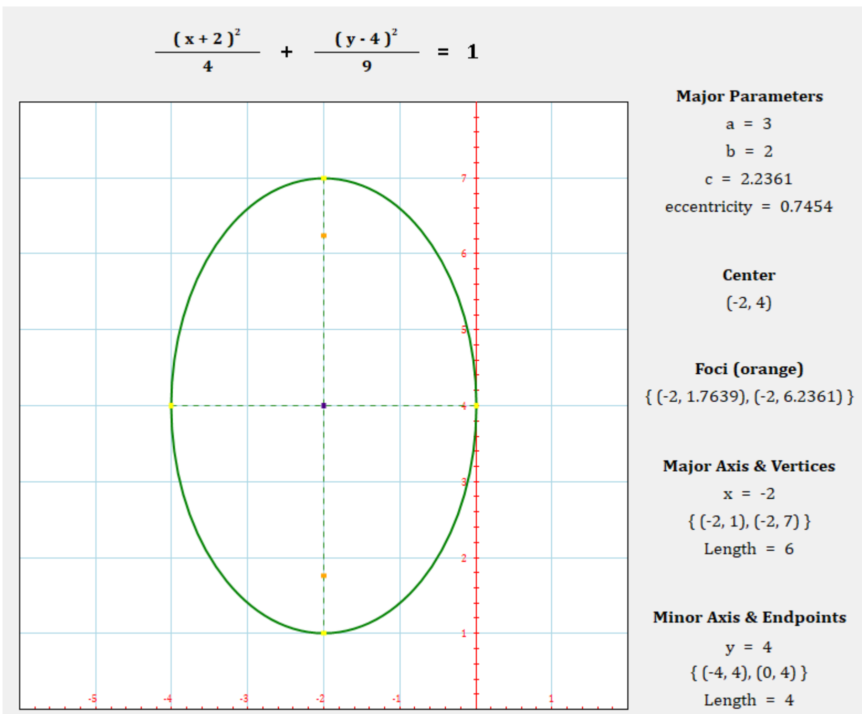
$$(h,k)=\left(-2,\frac{1+7}{2}\right)=(-2,4)$$

- Calculate a as half the distance between the major axis vertices. $a=\dfrac{7-1}{2}=3$
- Calculate b as half the distance between the minor axis vertices. $b=\dfrac{0-(-4)}{2}=2$
- Major axis vertices exist at $(h,\ k\pm a)=(-2,\ 4\pm 3)$ which matches the given values
- Minor axis vertices exist at $(h\pm b,\ k)=(-2\pm 2,\ 4)$ which matches the given values

So, the standard form for the ellipse defined above is:

$$\frac{(x-(-2))^2}{2^2}+\frac{(y-4)^2}{3^2}=1 \quad\Rightarrow\quad \boldsymbol{\frac{(x+2)^2}{4}+\frac{(y-4)^2}{9}=1}$$

We are not required to graph the ellipse, but here's what it looks like:



Write the first four terms of the sequence defined by the recursion formula.

16) $a_1 = 3$ and $a_n = 4a_{n-1} - 1$ for $n \geq 2$ 16) ____________

We want a_1 through a_4

$a_1 = 3$

$a_2 = 4(3) - 1 = 11$

$a_3 = 4(11) - 1 = 43$

$a_4 = 4(43) - 1 = 171$

The resulting terms, then, are: $\mathbf{3, 11, 43, 171}$

Write the first four terms of the sequence whose general term is given.

17) $a_n = \dfrac{3^n}{(n+1)!}$ 17) ____________

Let's write the terms as given, then simplify and reduce the sequence.

$$\frac{3^1}{(1+1)!}, \frac{3^2}{(2+1)!}, \frac{3^3}{(3+1)!}, \frac{3^3}{(4+1)!}$$

$$\frac{3}{2!}, \frac{9}{3!}, \frac{27}{4!}, \frac{81}{5!} \quad \Rightarrow \quad \frac{3}{2}, \frac{9}{6}, \frac{27}{24}, \frac{81}{120} \quad \Rightarrow \quad \mathbf{\frac{3}{2}, \frac{3}{2}, \frac{9}{8}, \frac{27}{40}}$$

Find the indicated sum.

18) $\displaystyle\sum_{i=7}^{10} \frac{1}{i-4}$ 18) ____________

This is the sum of a **harmonic sequence** because **the denominators of the terms form an arithmetic sequence**. The only method we know is to add the terms.

$$S = \frac{1}{7-4} + \frac{1}{8-4} + \frac{1}{9-4} + \frac{1}{10-4}$$

$$= \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$$

The common denominator will be the least common multiple of the denominators, i.e., 60

$$S = \left(\frac{20}{20} \cdot \frac{1}{3}\right) + \left(\frac{15}{15} \cdot \frac{1}{4}\right) + \left(\frac{12}{12} \cdot \frac{1}{5}\right) + \left(\frac{10}{10} \cdot \frac{1}{6}\right)$$

$$= \frac{20 + 15 + 12 + 10}{60} = \frac{57}{60} = \mathbf{\frac{19}{20}} = \mathbf{0.95}$$

The general term of a sequence is given. Determine whether the given sequence is arithmetic, geometric, or neither. If the sequence is arithmetic, find the common difference; if it is geometric, find the common ratio.

23) $a_n = 4n - 2$ 23) ____________

Look at the sequence. Letting $n = 1, 2, 3, 4, \ldots$

$2, 6, 10, 14, \ldots$

This is an **arithmetic sequence** because we are "adding" 4 to get each successive term. Also, not that **the common difference is what we are adding, i.e., 4**.

Use the formula for the sum of the first n terms of a geometric sequence to solve.

24) Find the sum of the first 8 terms of the geometric sequence: −8, −16, −32, −64, −128, ... 24) ____________

Method 1: Add 'em up (note that $r = 2$):

$$-8 - 16 - 32 - 64 - 128 - 256 - 512 - 1024 = \mathbf{-2,040}$$

Method 2: Use the geometric series sum formula: $S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right)$

$a_1 = -8 \qquad r = 2 \qquad n = 8$

$$S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right) = -8\left(\frac{2^8 - 1}{2 - 1}\right) = \frac{-8 \cdot 255}{1} = \mathbf{-2,040}$$

Convert the equation to the standard form for a parabola by completing the square on x or y as appropriate.

28) $x^2 - 2x + 7y - 34 = 0$ 28) ____________

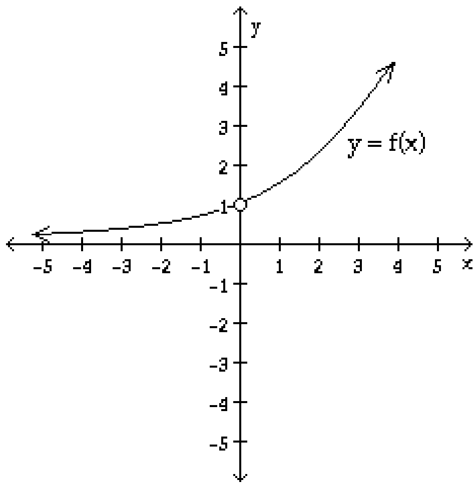
This is a parabola because we see an x^2 term, but no y^2 term. Standard form for the given equation, because there is an x^2 term, is:

$$(x - h)^2 = 4p(y - k)$$

Original Equation:	$x^2 - 2x + 7y - 34 = 0$	
Subtract $(7y - 34)$:	$x^2 - 2x$	$= -7y + 34$
Add $\left(\frac{2}{2}\right)^2 = 1$:	$x^2 - 2x + 1$	$= -7y + 35$
Simplify both sides:	$\mathbf{(x - 1)^2 = -7(y - 5)}$	

The graph of a function is given. Use the graph to find the indicated limit and finction value, or state that the limit or function value does not exist.

31) a. $\lim_{x \to 0} f(x)$ b. $f(0)$ 31) ____________



The limits from the left and right of 0 both exist and are equal. Clearly, $f(x)$ approaches the value of 1 from both the left and right as x approaches 0. Therefore,

$$\lim_{x \to 0} f(x) = 1$$

The value of $f(0)$ does not exist on the graph:

$f(0)$ does not exist because there is no value shown for $f(0)$ on the graph.

Note also that since $f(0)$ does not exist, the function is NOT continuous at $x = 1$.

Use properties of limits to find the indicated limit. It may be necessary to rewrite an expression before limit properties can be applied.

32) $\lim_{x \to -3} \frac{x^2 - 2x - 15}{x + 3}$ 32) ____________

For a rational expression, try **simplification** first.

$$\lim_{x \to -3} \frac{x^2 - 2x - 15}{x + 3} = \lim_{x \to -3} \frac{(x+3)(x-5)}{x+3} = \lim_{x \to -3} (x - 5) = (-3 - 5) = \mathbf{-8}$$

In this expression, there is a hole at $x = -3$. **In the case of a hole in a rational expression, a limit will exist but the function will not be continuous at the location of the hole.**

33) $\lim_{x \to 0} \frac{\sqrt{4 + x} - 2}{x}$ 33) ____________

Looks like a case of **Rationalize the Numerator.** Use the conjugate of the numerator for this purpose.

$$\lim_{x \to 0} \frac{\sqrt{4+x} - 2}{x} = \lim_{x \to 0} \frac{\sqrt{4+x} - 2}{x} \cdot \frac{\sqrt{4+x} + 2}{\sqrt{4+x} + 2} = \lim_{x \to 0} \frac{4 + x - 4}{x\left(\sqrt{4+x} + 2\right)}$$

$$= \lim_{x \to 0} \frac{x}{x\left(\sqrt{4+x} + 2\right)} = \lim_{x \to 0} \frac{1}{\left(\sqrt{4+x} + 2\right)} = \frac{1}{\left(\sqrt{4} + 2\right)} = \mathbf{\frac{1}{4}}$$

34) $\lim\limits_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$ 34) ____________

We can either rationalize the numerator or factor the denominator. I factor the denominator.

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{(x - 4)} = \lim_{x \to 4} \frac{(\sqrt{x} - 2)}{(\sqrt{x} - 2)(\sqrt{x} + 2)} = \lim_{x \to 4} \frac{1}{(\sqrt{x} + 2)} = \frac{1}{(\sqrt{4} + 2)} = \frac{\mathbf{1}}{\mathbf{4}}$$

35) $\lim\limits_{x \to 2} \frac{x^2 - 4}{x^3 - 8}$ 35) ____________

For a rational expression, try **simplification** first.

$$\lim_{x \to 2} \frac{x^2 - 4}{x^3 - 8} = \lim_{x \to 2} \frac{(x - 2)(x + 2)}{(x - 2)(x^2 + 2x + 4)} = \lim_{x \to 2} \frac{(x + 2)}{(x^2 + 2x + 4)} = \frac{2 + 2}{2^2 + (2 \cdot 2) + 4} = \frac{4}{12} = \frac{\mathbf{1}}{\mathbf{3}}$$

Determine for what numbers, if any, the given function is discontinuous.

36) $f(x) = \begin{cases} x - 5 & \text{if } x \le 5 \\ x^2 - 10 & \text{if } x > 5 \end{cases}$ 36) ____________

To be discontinuous, the limits from the left and right need to be unequal. Polynomials are continuous everywhere. So, the only possible point of discontinuity is at $x = 5$, i.e., at the split between the two parts of the function. Let's check for continuity at $x = 5$.

a) Limit from the left:

$$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} (x - 5) = (5 - 5) = 0$$

b) Limit from the right:

$$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} (x^2 - 10) = 5^2 - 10 = 15$$

c) Overall limit:

Since $\lim\limits_{x \to 5^-} f(x) \neq \lim\limits_{x \to 5^+} f(x)$, (i.e., $0 \neq 15$), we know that $\lim\limits_{x \to 5^+} f(x)$ does not exist.

Therefore, we conclude that $\boldsymbol{f(x)}$ **is not continuous at** $\boldsymbol{x = 5}$.

37) $f(x) = \frac{2x + 5}{x^2 - 4}$ 37) ____________

Rational functions are discontinuous when the denominator is zero, i.e. at $\boldsymbol{x = \pm 2}$.

Find the slope of the tangent line to the graph of f at the given point.

38) $f(x) = -4x^2 + 7x$ at $(5, -65)$ 38) ____________

The slope of the tangent line is obtained by taking a derivative of $f(x)$.

$f(x) = -4x^2 + 7x$

$f'(x) = -8x + 7$

$f'(5) = -8(5) + 7 = \mathbf{-33}$

39) $f(x) = x^2 + 11x - 15$ at $(1, -3)$ 39) ____________

The slope of the tangent line is obtained by taking a derivative of $f(x)$.

$f(x) = x^2 + 11x - 15$

$f'(x) = 2x + 11$

$f'(1) = 2(1) + 11 = \mathbf{13}$

Find the slope-intercept equation of the tangent line to the graph of f at the given point.

40) $f(x) = x^2 + 5x$ at $(4, 36)$ 40) ____________

The equation of a tangent line requires a point and a slope. We are given the point $(4, 36)$, but we need a slope, which we get by taking a derivative of $f(x)$.

$f(x) = x^2 + 5x$

$f'(x) = 2x + 5$

$f'(4) = 2(4) + 5 = 13$, which is the slope of the tangent line at $(4, 36)$.

Then, using this slope and the point we are given for this problem, the equation of the tangent line is:

$y = 13(x - 4) + 36$ (in h, k form)

$y = 13x - 52 + 36$

$\boldsymbol{y = 13x - 16}$ (in slope-intercept form)

41) $f(x) = \sqrt{x}$ at (16, 4) 41) ___________

The equation of a tangent line requires a point and a slope. We are given the point $(16, 4)$, but we need a slope, which we get by taking a derivative of $f(x)$.

$$f(x) = \sqrt{x} = x^{1/2}$$

$$f'(x) = \frac{1}{2}x^{\left(\frac{1}{2}-1\right)} = \frac{1}{2}x^{(-1/2)} = \frac{1}{2\sqrt{x}}$$

$f'(16) = \frac{1}{2\sqrt{16}} = \frac{1}{8}$, which is the slope of the tangent line at $(16, 4)$.

Then, using this slope and the point we are given for this problem, the equation of the tangent line is:

$y = \frac{1}{8}(x - 16) + 4$ (in h, k form)

$$y = \frac{1}{8}x - 2 + 4$$

$\boldsymbol{y = \frac{1}{8}x + 2}$ (in slope-intercept form)

Find the derivative of f at x. That is, find f '(x).

42) $f(x) = \frac{-4}{x}$; $x = 4$ 42) ___________

$$f(x) = -\frac{4}{x} = -4x^{-1}$$

$$f'(x) = (-1)(-4x^{-1-1}) = 4x^{-2} = \frac{4}{x^2}$$

$$f'(4) = \frac{4}{4^2} = \boldsymbol{\frac{1}{4}}$$

The equations given in problems 43 and 44 are incorrect for the situations described. The general equation for this kind of problem should be $s(t) = -16t^2 + v_0 t + s_0$, where v_0 is the initial velocity of the object and s_0 is the initial position of the object. In order to do these problems, we need to ignore what the paragraph says about the situation and focus on the given equation.

Solve the problem.

43) An explosion causes debris to rise vertically with an initial velocity of 6 feet per second. The function s(t) = $-16t^2 + 96t$ describes the height of the debris above the ground, s(t), in feet, t seconds after the explosion. What is the instantaneous velocity of the debris when it hits the ground? 43) ____________

$s(t) = -16t^2 + 96t$

The debris hits the ground (all at once according to this problem) when $s(t) = 0$. Let's find the value of t for when the debris hits the ground.

$s(t) = -16t^2 + 96t = 0$

$s(t) = -16t(t-6) = 0$

$t = \{0, 6\}$

The explosion happens at time $t = 0$, so the debris hits the ground at $t = 6$.

Recall that **instantaneous velocity is the derivative of position**.

$v(t) = s'(t) = -32t + 96$ This is the velocity function.

$v(6) = -32 \cdot (6) + 96 =$ **-96 ft. per second.**

Note: the negative sign in the answer means that the object is falling, not rising.

44) An explosion causes debris to rise vertically with an initial velocity of 3 feet per second. The function s(t) = $-16t^2 + 48t$ describes the height of the debris above the ground, s(t), in feet, t seconds after the explosion. What is the instantaneous velocity of the debris 1.2 second(s) after the explosion? 44) ____________

$s(t) = -16t^2 + 48t$

Recall that **instantaneous velocity is the derivative of position**. We want velocity at $t = 1.2$.

$v(t) = s'(t) = -32t + 48$ This is the velocity function.

$v(1.2) = -32 \cdot (1.2) + 48 =$ **9.6 ft. per second.**

Note: the answer is positive, indicating that object is still rising 1.2 seconds after the explosion.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Complete the identity.

45) $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = ?$ 45) ______

A) sec x csc x B) $-2 \tan^2 x$ C) sin x tan x D) 1 + cot x

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin x}{\sin x} \cdot \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x \cdot \cos x}$$

$$= \frac{1}{\sin x \cdot \cos x}$$

$$= \frac{1}{\sin x} \cdot \frac{1}{\cos x}$$

$$= \mathbf{\csc x \cdot \sec x}$$

Answer A

47) $\frac{\cos x - \sin x}{\cos x} + \frac{\sin x - \cos x}{\sin x} = ?$ 47) ______

A) 2 − sec x csc x B) sec x csc x C) 1 − sec x csc x D) 2 + sec x csc x

$$\frac{(\cos x - \sin x)}{\cos x} + \frac{(\sin x - \cos x)}{\sin x} = \left(1 - \frac{\sin x}{\cos x}\right) + \left(1 - \frac{\cos x}{\sin x}\right)$$

$$= 2 - \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$= 2 - \left(\frac{\sin x}{\sin x} \cdot \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x}\right)$$

$$= 2 - \left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right)$$

$$= 2 - \left(\frac{1}{\sin x \cos x}\right)$$

$$= 2 - \left(\frac{1}{\sin x} \cdot \frac{1}{\cos x}\right)$$

$$= \mathbf{2 - \csc x \sec x}$$

Answer A

48) $\cos(\alpha + \beta) + \cos(\alpha - \beta) = ?$ 48) ______

A) $\sin \beta \cos \alpha$ B) $2\sin \alpha \cos \beta$ C) $2\cos \alpha \cos \beta$ D) $\cos \alpha \cos \beta$

$$\cos(\alpha + \beta) + \cos(\alpha - \beta) = (\cos \alpha \cos \beta - \sin \alpha \sin \beta) + (\cos \alpha \cos \beta + \sin \alpha \sin \beta)$$

$$= \mathbf{2 \cos \alpha \cos \beta}$$

Answer C

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find the exact value by using a sum or difference identity.

52) $\sin(215° - 95°)$ 52) ____________

$$\sin(215° - 95°) = \sin(120°) = \sin(60°) = \frac{\sqrt{3}}{2}$$

53) $\sin 165°$ 53) ____________

$$\sin(165°) = \sin(120° + 45°)$$

$$= \sin 120° \cos 45° + \cos 120° \sin 45°$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{-1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

54) $\cos 285°$ 54) ____________

$$\cos(285°) = \cos(150° + 135°)$$

$$= \cos 150° \cos 135° - \sin 150° \sin 135°$$

$$= \frac{-\sqrt{3}}{2} \cdot \frac{-\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

Find the exact value of the expression.

55) $\sin 265° \cos 25° - \cos 265° \sin 25°$ 55) ____________

$$\sin 265° \cos 25° - \cos 265° \sin 25° = \sin(265° - 25°) = \sin(240°) = -\sin(60°) = -\frac{\sqrt{3}}{2}$$

56) $\sin \frac{2\pi}{9} \cos \frac{\pi}{18} - \sin \frac{\pi}{18} \cos \frac{2\pi}{9}$ 56) ____________

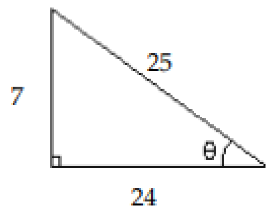
$$\sin \frac{2\pi}{9} \cos \frac{\pi}{18} - \cos \frac{2\pi}{9} \sin \frac{\pi}{18} = \sin\left(\frac{2\pi}{9} - \frac{\pi}{18}\right) = \sin\left(\frac{3\pi}{18}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

57) $\sin 185° \cos 65° - \cos 185° \sin 65°$ 57) ____________

$$\sin 185° \cos 65° - \cos 185° \sin 65° = \sin(185° - 65°) = \sin(120°) = \sin(60°) = \frac{\sqrt{3}}{2}$$

Use the figure to find the exact value of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$ **.**

58) 58) ____________

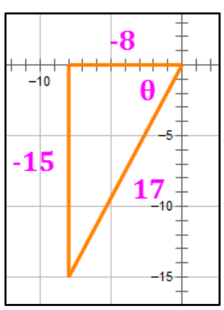


$$\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{7}{25}\cdot\frac{24}{25} = \mathbf{\frac{336}{625}}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{24}{25}\right)^2 - \left(\frac{7}{25}\right)^2 = \frac{576-49}{625} = \mathbf{\frac{527}{625}}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\frac{336}{625}}{\frac{527}{625}} = \mathbf{\frac{336}{527}}$$

61) $\tan\theta = \frac{15}{8}$, θ lies in quadrant III 61) ____________



$$\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\left(-\frac{15}{17}\right)\cdot\left(-\frac{8}{17}\right) = \mathbf{\frac{240}{289}}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(-\frac{8}{17}\right)^2 - \left(-\frac{15}{17}\right)^2 = -\mathbf{\frac{161}{289}}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = -\mathbf{\frac{240}{161}}$$

Write the expression as the sine, cosine, or tangent of a double angle. Then find the exact value of the expression.

62) $2\sin 120°\cos 120°$ 62) ____________

$$2\sin 120°\cos 120° = \sin(2\cdot 120°) = \mathbf{\sin 240°} = -\mathbf{\frac{\sqrt{3}}{2}}$$

63) $\dfrac{2\tan\frac{5\pi}{8}}{1-\tan^2\frac{5\pi}{8}}$ 63) ____________

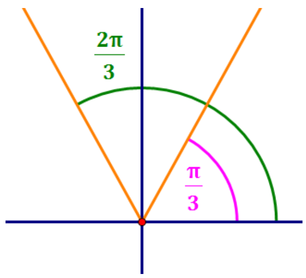
$$\frac{2\tan\frac{5\pi}{8}}{1-\tan^2\frac{5\pi}{8}} = \tan\left(2\cdot\frac{5\pi}{8}\right) = \mathbf{\tan\left(\frac{5\pi}{4}\right) = 1}$$

Solve the equation on the interval $[0, 2\pi)$.

66) $\sin 4x = \dfrac{\sqrt{3}}{2}$ 66) ______________

When working with a problem in the interval $[0, 2\pi)$ that involves a function of kx, it is useful to expand the interval to $[0, 2k\pi)$ for the first steps of the solution.

So, we want all solutions to $\sin u = \frac{\sqrt{3}}{2}$ where $u = 4x$ is an angle in the interval $[0, 8\pi)$. Note that, beyond the two solutions suggested by the diagram, additional solutions are obtained by adding multiples of 2π to those two solutions.



Note that there are 8 solutions because the usual number of solutions (i.e., 2) is increased by a factor of $k = 4$.

Using the diagram at left, we get the following solutions:

$$u = 4x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}, \frac{19\pi}{3}, \frac{20\pi}{3}$$

Then, dividing by 4, we get:

$$x = \frac{\pi}{12}, \frac{2\pi}{12}, \frac{7\pi}{12}, \frac{8\pi}{12}, \frac{13\pi}{12}, \frac{14\pi}{12}, \frac{19\pi}{12}, \frac{20\pi}{12}$$

And simplifying, we get:

$$\boldsymbol{x = \frac{\pi}{12}, \frac{\pi}{6}, \frac{7\pi}{12}, \frac{2\pi}{3}, \frac{13\pi}{12}, \frac{7\pi}{6}, \frac{19\pi}{12}, \frac{5\pi}{3}}$$

69) $\cos x = \sin x$ 69) ______________

This problem is most easily solved by inspection. Where are the cosine and sine functions equal? At the angles with a reference angle of $\frac{\pi}{4}$ in Q1 and Q3.

Therefore, $\boldsymbol{x = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}}$

Another method that can be used to solve this kind of problem is shown in the solution to the next problem.

70) $\sin^2 x - \cos^2 x = 0$ 70) ____________

$$(\sin x + \cos x)(\sin x - \cos x) = 0$$

$(\sin x + \cos x) = 0$ or $(\sin x - \cos x) = 0$

$\sin x = -\cos x$ $\quad\quad$ $\sin x = \cos x$

$\tan x = -1$ $\quad\quad$ $\tan x = 1$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$ $\quad\quad$ $x = \frac{\pi}{4}, \frac{5\pi}{4}$

$$\boldsymbol{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$$

In this problem, we take a different approach to solving $\sin x = \cos x$, which could, as in Problem 65, above, be solved by inspection. Since $\sin x$ and $\cos x$ are never both zero, we can divide both sides by $\cos x$ to get the resulting $\tan x$ equations.

71) $\sin^2 x + \sin x = 0$ 71) ____________

$$\sin x \ (\sin x + 1) = 0$$

$\sin x = 0$ or $(\sin x + 1) = 0$

$x = 0, \pi$ $\quad\quad$ $\sin x = -1$

$x = \frac{3\pi}{2}$

$$\boldsymbol{x = 0, \pi, \frac{3\pi}{2}}$$

Solve the equation on the interval $[0, 2\pi)$.

72) $\tan^2 x \sin x = \tan^2 x$ 72) ____________

$$\tan^2 x \sin x - \tan^2 x = 0$$

$$\tan^2 x\,(\sin x - 1) = 0$$

Be extra careful when dealing with functions other than sine and cosine, because there are values at which these functions are undefined.

$\tan x = 0$ or $(\sin x - 1) = 0$

$x = 0, \pi$ $\quad\quad$ $\sin x = 1$

$x = \frac{\pi}{2}$

While $x = \frac{\pi}{2}$ is a solution to the equation $\sin x = 1$, $\tan x$ is undefined at $x = \frac{\pi}{2}$, so $\frac{\pi}{2}$ is <u>not</u> a solution to this equation.

$$\boldsymbol{x = 0, \pi}$$

73) cos x + 2 cos x sin x = 0

73) ____________

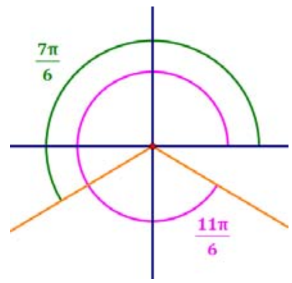
$\cos x\ (1 + 2\sin x) = 0$

$\cos x = 0$ or $(1 + 2\sin x) = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$



$$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$$

Solve the equation on the interval [0, 2π).

74) $\cos 2x = \sqrt{2} - \cos 2x$

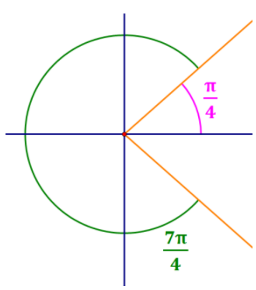
74) ____________

$2\cos 2x = \sqrt{2}$

$\cos 2x = \frac{\sqrt{2}}{2}$

Recall that working with a problem in the interval $[0, 2\pi)$ that involves a function of kx, it is useful to expand the interval to $[0, 2k\pi)$ for the first steps of the solution.

So, we want all solutions to $\cos u = \frac{\sqrt{2}}{2}$ where $u = 2x$ is an angle in the interval $[0, 4\pi)$. Note that, beyond the two solutions suggested by the diagram, additional solutions are obtained by adding 2π to those two solutions.



Note that there are 4 solutions because the usual number of solutions (i.e., 2) is increased by a factor of $k = 2$.

Using the diagram at left, we get the following solutions:

$$u = 2x = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4}$$

Then, dividing by 2, we get:

$$x = \frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}$$

We cannot simplify these answers any further.

75) $2\cos^2 x + \sin x - 2 = 0$ 75) ________

$2\cos^2 x + \sin x - 2 = 0$

$2\,(1 - \sin^2 x) + \sin x - 2 = 0$

$2 - 2\sin^2 x + \sin x - 2 = 0$

$-2\sin^2 x + \sin x = 0$

$\sin x\,(-2\sin x + 1) = 0$

When an equation contains more than one function, try to convert it to one that contains only one function.

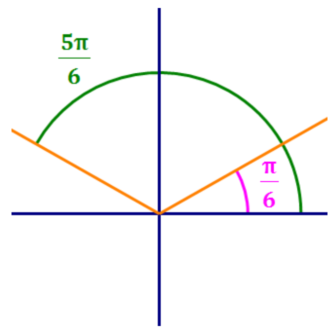
$\sin x = 0$ or $(-2\sin x + 1) = 0$

$x = 0, \pi$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$$\boldsymbol{x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi}$$



Use a calculator to solve the equation on the interval [0, 2π). Round the answer to two decimal places.

77) $\cos x = 0.74$ 77) ________

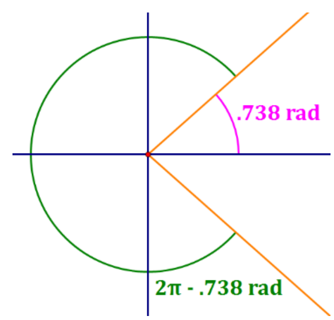
$\cos x = .74$

$x = 0.737726$ radians (by calculator)

$x = 2\pi - .737726$

$= 6.283185 - .737726 = 5.545459$ radians

Rounding to 2 decimal places gives: $\boldsymbol{x = \{.74, 5.55\}}$



Solve the equation on the interval $[0, 2\pi)$.

78) $\sin 2x - \sin x = 0$

78) ____________

$$\sin 2x - \sin x = 0$$

$$2 \sin x \cos x - \sin x = 0$$

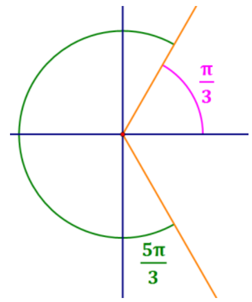
$$\sin x\,(2\cos x - 1) = 0$$

$\sin x = 0$ or $(2\cos x - 1) = 0$

$x = 0, \pi$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$



$$\boldsymbol{x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}}$$

If the question asked you to round to the nearest hundredth of a radian: $x = \{0, 1.05, 3.14, 5.24\}$